Phase Space Integration for Initial State Radiation In Hadronic Interaction

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Phase Space Integration for Initial State Radiation In Hadronic Interaction

- Introduction
- QCD Soft and Collinear Singularities
- DGLAP Splitting Functions in the Initial State Radiation
- Probability Distribution in the Initial State Radiation
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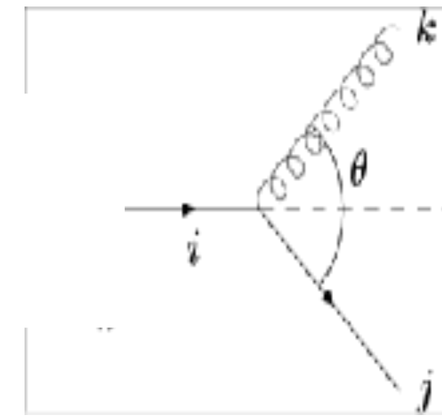
- Phase space generation
- Conclusion

Introduction

- Phase space integration is very important for collider experiments calculations, we want to generate an efficient phase space integration using the Altarelli-Parisi splitting functions as the underlying probability.

- For large numbers of partons we treat the regions where the emission of QCD radiation is enhanced, collinear parton splitting or soft (low-energy) gluon emission.

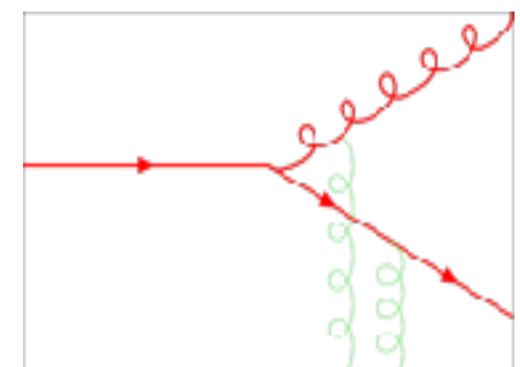
QCD Collinear and Soft Singularities



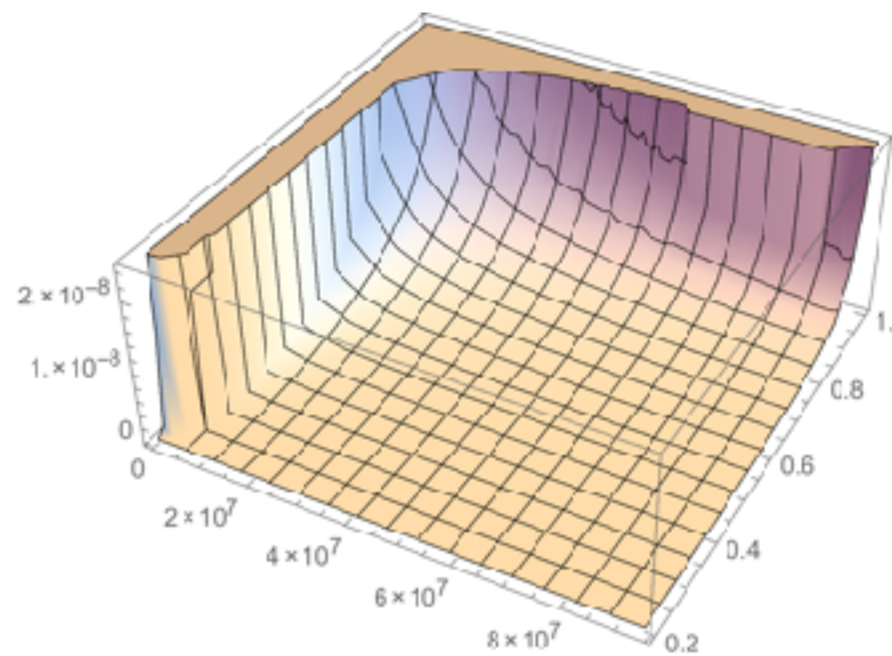
In the collinear limit the cross section for a process factorizes

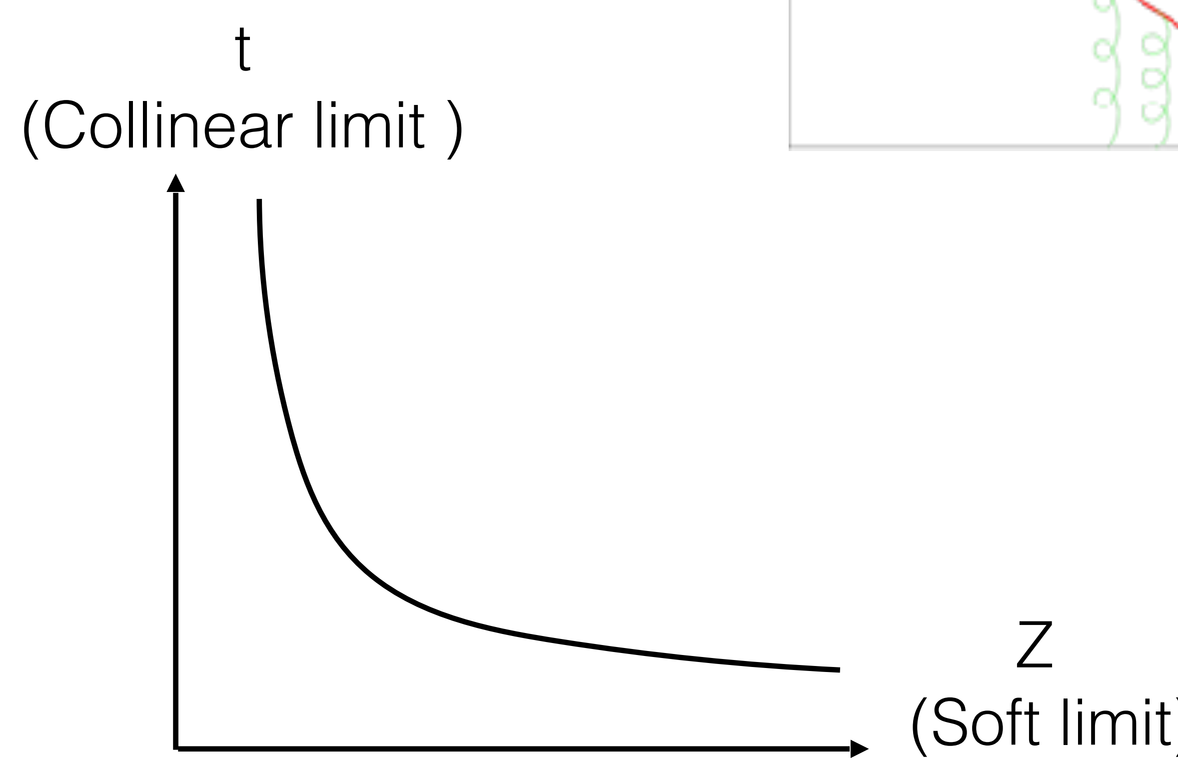
$$f_{a\to bc}(t,z)=\frac{\alpha_s}{2\pi}\frac{1}{t}P_{a\to bc}(z)\quad\frac{1}{z}\frac{f_a(t,x')}{f_a(t,x')}\theta(z_{min}<z<z_{max})$$

This expression is singular as t $\longrightarrow$ 0 (Collinear)



Soft gluons (low energy) come from all over the event.



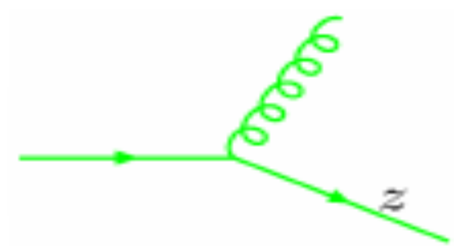


$P_b(x;t,z)$ Probability Distribution in 3D

DGLAP Functions in the Initial State Radiation

Initial state radiation ISR arises because incoming charged particles can radiate before entering the hard process.

The branching of these partons terminates when they collide to initiate the hard subprocess.



We have 7 Splitting functions

$$f_{a\to bc}(t,z)=\frac{\alpha_s}{2\pi}\,\frac{1}{t}\,P_{a\to bc}(z)\quad\frac{1}{z}\,\frac{f_a(t,x')}{f_a(t,x')}\,\theta(z_{min}<z<z_{max})$$

In the Initial state radiation

$$f_b(t,z)=\frac{\alpha_s}{2\pi}\,\frac{1}{t}\sum_{a,b}P_{a\to bc}(z)\quad\frac{1}{z}\,\frac{f_a(t,x')}{f_a(t,x')}\,\theta(z_{min}<z<z_{max})$$

Splitting kernels

$$P_{q\to qg}(z) = C_F\frac{1+z^2}{1-z}$$

$$P(z)_{q\to gq}=C_F\frac{1+(1-z)^2}{z}$$

$$P_{g\to q\bar{q}}(z) = T_R\left[z^2+(1-z)^2\right]$$

$$P_{g\to gg}(z) = C_A\left[\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right]$$

Splitting Probability in the Initial State Radiation

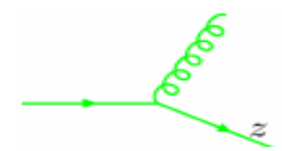
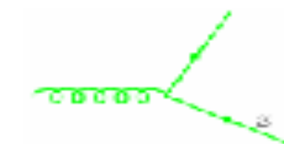
- We consider q, $\bar{q}$, g Splitting Probability

$$P_b(x;t,z,\varphi) = \sum_{a,b} f_{a\to bc}(t,z)\, \Delta_b(x;t,t_{max})$$

- Sudakov form factor :A given parton can only branch once, if it did not already do so

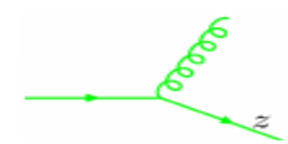
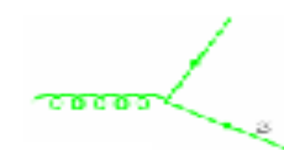
$$\Delta_b(x;t,t_{max}) = exp\left\{-\int_t^{t_{max}} dt' \int dz\, f_a(t,z)\right\}$$

•Quark splitting probability

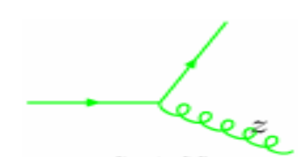
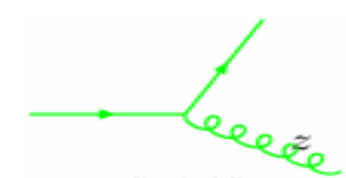
$$P_q(x;t,z) = [f_{qqg}(t,z) + f_{gq\bar{q}}(t,z)]\, \Delta_{qqg}(x;t,t_{max})\Delta_{gq\bar{q}}(x;t,t_{max})$$

•Antiquark splitting probability

$$P_{\bar{q}}(x;t,z) = [f_{\bar{q}\bar{q}g}(t,z) + f_{g\bar{q}q}(t,z)]\, \Delta_{\bar{q}\bar{q}g}(x;t,t_{max})\Delta_{g\bar{q}q}(x;t,t_{max})$$

•Gluon splitting probability

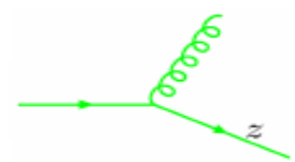




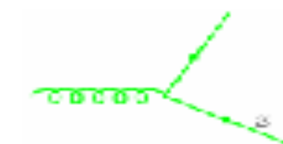
$$P_g(x;t,z) = [f_{ggg}(t,z) + f_{qgq}(t,z) + f_{\bar{q}g\bar{q}}(t,z)]\, \Delta_{ggg}(x;t,t_{max})\Delta_{qgq}(x;t,t_{max})\Delta_{\bar{q}g\bar{q}}(x;t,t_{max})$$

Probability Distribution

Difficult to distribute according to these functions all the way to t = 0. Instead :
Distribute according to P(x,t,z) for $t > t_{IR}$,
and according to a flat distribution for $t < t_{IR}$

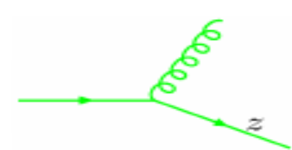
- Quark distribution

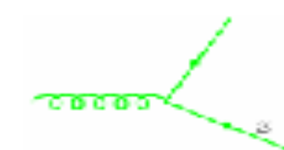




$$\int_{t_{IR}}^{t_{max}}\int_{z_{min}}^{z_{max}} dtdz P_g(x;t,z) = 1 - [\Delta_{ggg}(x;t_{IR},t_{max})\Delta_{qgq}(x;t_{IR},t_{max})\Delta_{\bar{q}g\bar{q}}(x;t_{IR},t_{max})]$$

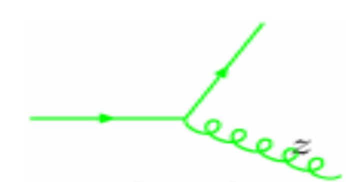
- Antiquark distribution





$$\int_{t_{IR}}^{t_{max}}\int_{z_{min}}^{z_{max}} dtdz P_{\bar{q}}(x;t,z) = 1 - [\Delta_{\bar{q}\bar{q}g}(x;t_{IR},t_{max})\Delta_{g\bar{q}q}(x;t_{IR},t_{max})]$$

- Gluon Quark distribution



$$\int_{t_{IR}}^{t_{max}}\int_{z_{min}}^{z_{max}} dtdz P_q(x;t,z) = 1 - [\Delta_{qqg}(x;t_{IR},t_{max})\Delta_{gq\bar{q}}(x;t_{IR},t_{max})]$$

Monte Carlo Procedure

P(x,t,z) depends on t, z and x, we cannot calculate Sudakov factor analytically.

$$\int_{t_{IR}}^{t_{max}}\int_{z_{min}}^{z_{max}} dtdz P_q(x;t,z) = 1 - [\Delta_{qqg}(x;t_{IR},t_{max})\Delta_{gq\bar{q}}(x;t_{IR},t_{max})]$$

Encode the dependence of P(x; t, z) on t, z and x into a grid. The **algorithm** for determining the x, z, and t values for an ISR emission will be as follows:

1.It requires a 3-dimensional grid for the probability P(s; v; r).
2. Determine the value of r that corresponds to the value of x for which we seek the ISR
3.we will have a set of grids describing the probability distribution P(s; v) for different values of r.
4. Map the values of s and v on a values of t and z
5. Distribute the value of s and v according to that grid

Transformations to calculate the Grid

we write the functional dependence of r, s and v on x, t and z if it is logarithmic or another form.

$$r(x) = 1 - \frac{\ln(x)}{\ln(xmin)} \qquad s(t) = 1 - \frac{\ln(t/t_{\max})}{\ln(t_{\mathrm{IR}}/t_{\max})}$$

q to qg splitting

$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z} \qquad v(t,z) = \frac{\ln \frac{1-z_{\min}(t)}{1-z}}{\ln \frac{1-z_{\min}(t)}{1-z_{\max}(t)}}$$

g to gg splitting

$$P_{g\to gg}(z) = C_A \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right] \qquad v(t,z) = \frac{\ln \frac{z(1-z_{\min}(t))}{z_{\min}(t)(1-z)}}{\ln \frac{z_{\max}(t)(1-z_{\min}(t))}{z_{\min}(t)(1-z_{\max}(t))}}$$

g to q$\bar{q}$ splitting

$$P_{g\to q\bar{q}}(z) = T_R\left[z^2 + (1-z)^2\right] \qquad v(t,z) = \frac{z - z_{\min}(t)}{z_{\max}(t) - z_{\min}(t)}$$

q to gq splitting

$$P(z)_{q\to gq} = C_F \frac{1+(1-z)^2}{z} \qquad v(t,z) = \frac{\ln\left(\frac{z}{z_{\min}(t)}\right)}{\ln\left(\frac{z_{\max}(t)}{z_{\min}(t)}\right)}$$

Monte Carlo Calculation
Distribution according to the grids

- The probability density P(x,t,z) is a function of n-component , related to the momenta of the particles involved, and we want to integrate it over some region in x,t,z space of volume V

- The variance can be reduced by a change of variables that "flattens" the integrand using the Jacobian $Jac(x(r),t(s,r),z(s,r,v))$.

- Write the phase space volume as an n- dimensional hypercube with volume 1

$$\int_{x_{min}}^{x_{max}}\int_{t_{IR}}^{t_{max}}\int_{z_{min}}^{z_{max}} dxdtdz\, P_b(x;t,z) = \int_0^1\int_0^1\int_0^1 drdsdv\, Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$$

- This is a function with less variance than P(x,t,z) itself, then the error will be reduced by distributing points uniformly in r,s,v space

$$G_{ijk}=\frac{dxdtdz}{drdsdv}(x_{ijk},t_{ijk},z_{ijk})P(x_{ijk},t_{ijk},z_{ijk})$$

Monte Carlo calculation and the grid version of the probability distribution function

If n points {xk, ti, zj, where i j k=1,…,n} are distributed randomly in V ,then use the central limit theorem of statistics to get the mean value of P(x,t,z) on those points which is an estimator of the integral,

$$P_{grid}\ (t > t_{IR}) = \frac{1}{n_k n_i n_j} \sum_{ijk} G_{ijk}$$

- We calculated the grid version of the splitting probabilities each grid is 10x10x10 dimensions

Histories and Signatures

- Every grid is representing one history , in total 6 signatures: each signature rely on one or more than one history

- $\bar{q}\, q \to zg$: $\bar{q}\,(q \to qg) \to zg + (\bar{q} \to \bar{q}\, g)q \to zg$
- $q\, \bar{q} \to zg$: $q\,(\bar{q} \to \bar{q}\, g) \to zg + (q \to q\, g)\bar{q} \to zg$
- $qg \to zq$: $q\,(g \to \bar{q}\, q) \to zq$
- $\bar{q}\, g \to z\bar{q}$: $\bar{q}(g \to q\, \bar{q}) \to z\bar{q}$
- $gq \to zq$: $(g \to \bar{q}\, q)q \to zq$
- $g\bar{q} \to z\bar{q}$: $(g \to q\, \bar{q})\, \bar{q} \to z\bar{q}$

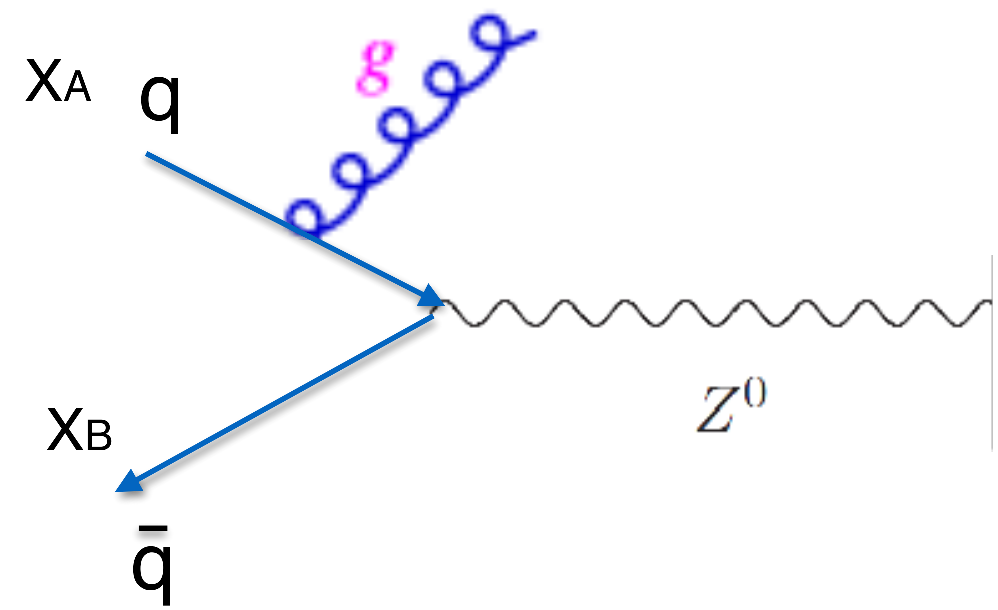
History= Feynman diagram of signature

Signature= incoming and outgoing particles in one interaction

$$q\ \bar{q} \longrightarrow gz$$

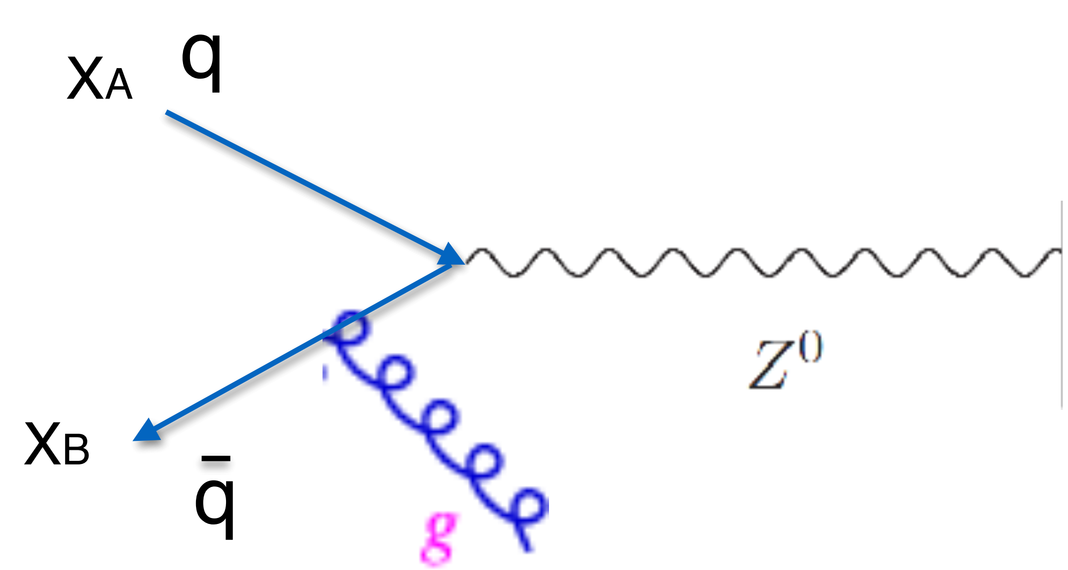
$$=$$

$$(q \longrightarrow q\ g)\bar{q} \longrightarrow zg$$



$$+$$

$$q\ (\bar{q} \longrightarrow \bar{q}\ g) \longrightarrow zg$$



Tests of the grids

A/ All the grids calculated for one type of particles are summing up to one :

$Pq=Pg \longrightarrow q\,\bar{q} + Pq \longrightarrow q\, g$

$P\bar{q}=Pg \longrightarrow \bar{q}q + P\bar{q} \longrightarrow \bar{q}\, g$

$Pg=Pg \longrightarrow gg + Pq \longrightarrow gq + P\bar{q} \longrightarrow g\bar{q}$

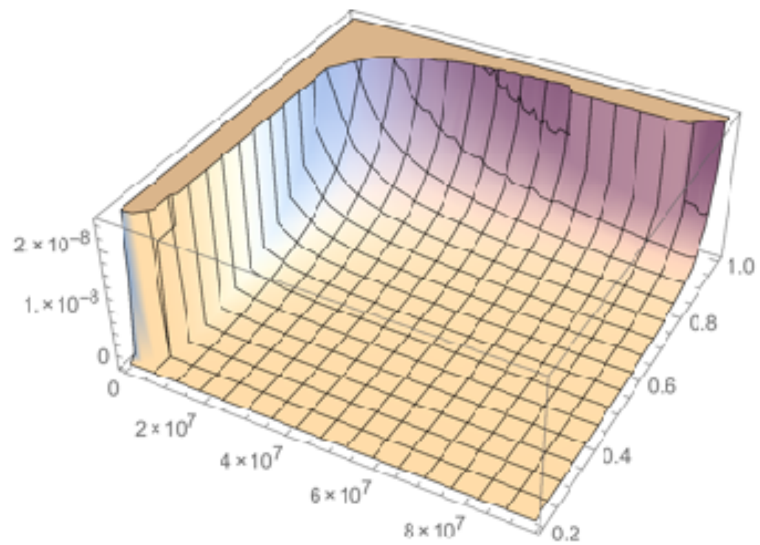
in total we have 31 grid, considering the type of particle going into the hard process there are only 11 grid 5Pq, 5P$\bar{q}$, 1Pg

B/ Plot all the piecewise functions, which is the grid version of P(t, z, x) in the same plot, the results were showing a big agreement between all the plots.

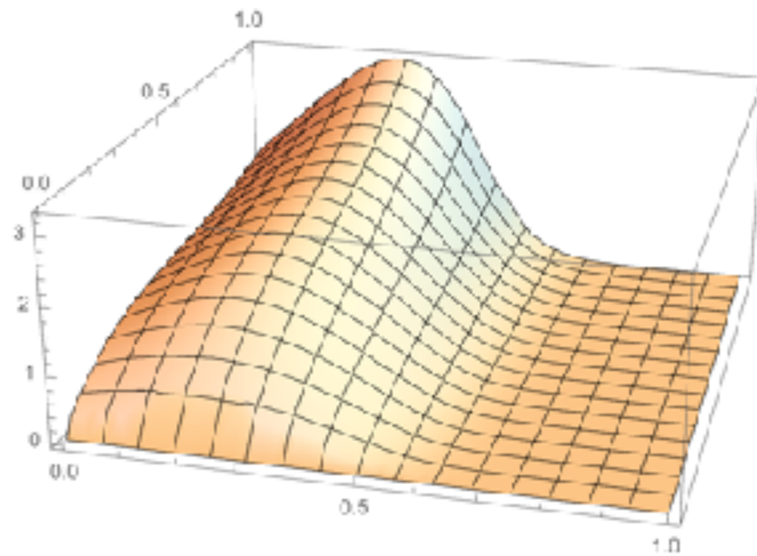
Up Quark distribution function

for all the 11 grids

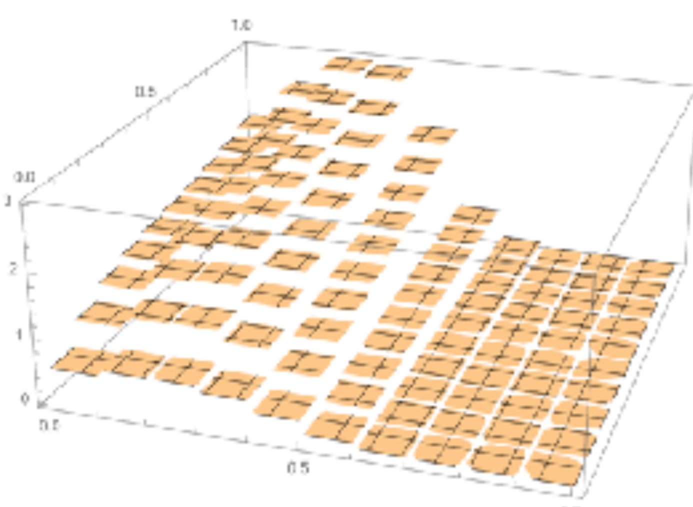
$P_b(x;t,z)$

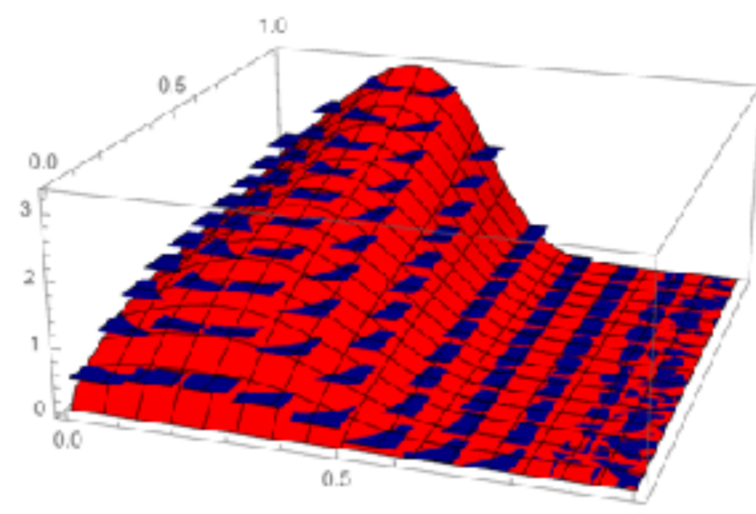


$Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$



- We have five grids for the Quark distribution





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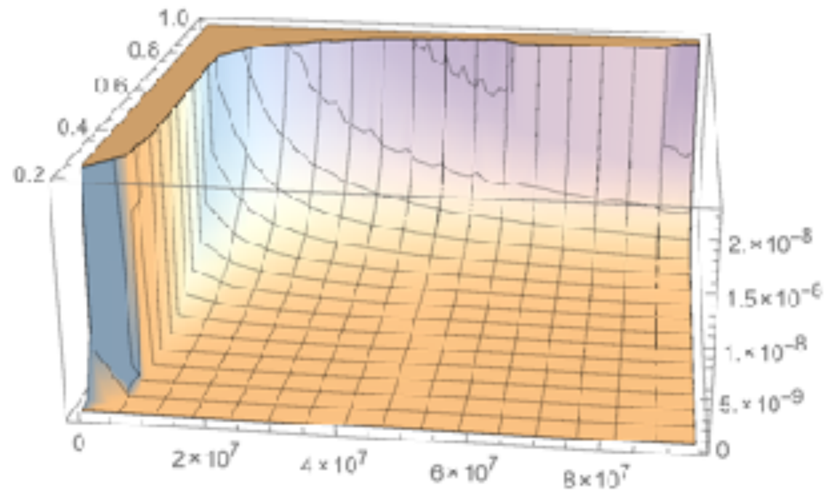
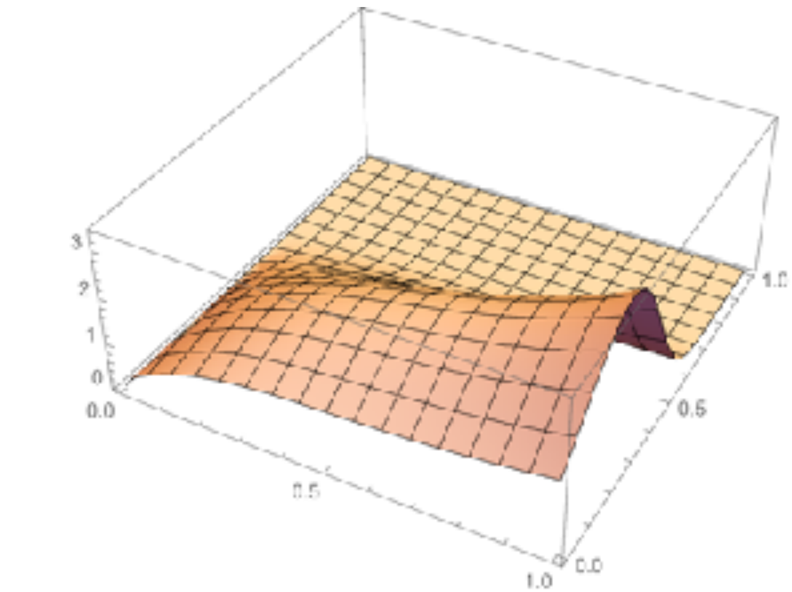
G_{ijk}

Anti Up Quark distribution function

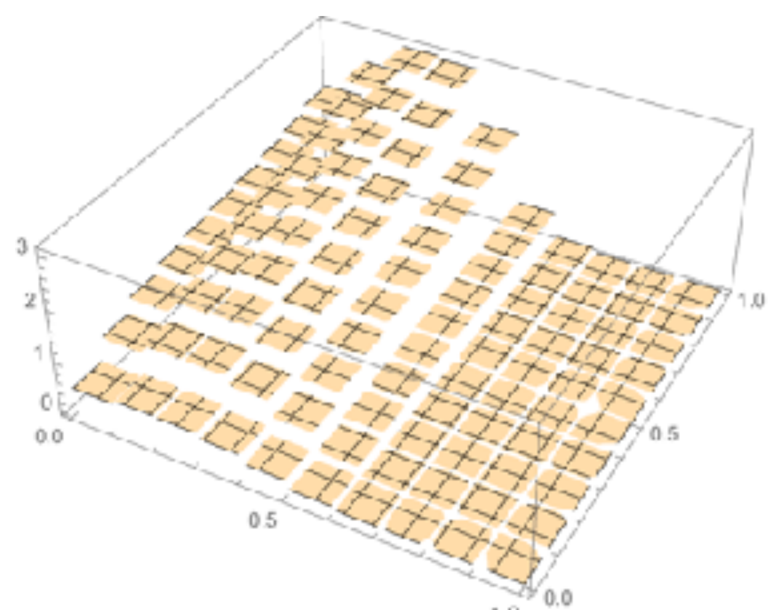
$P_b(x;t,z)$



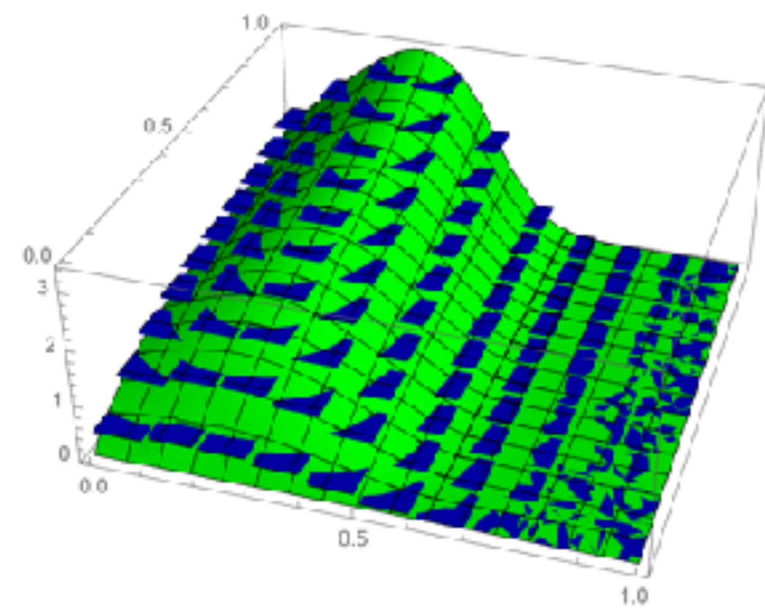
$Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$



- We have five grids for the Antiquark distribution

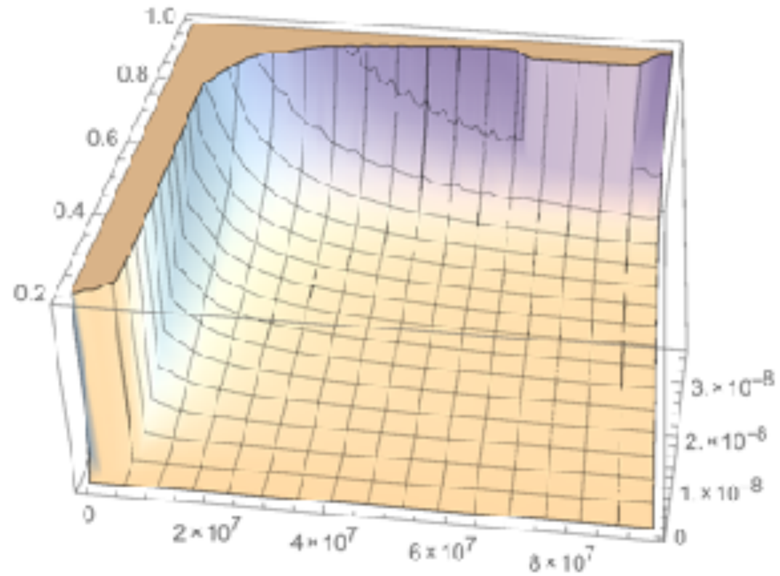


G_{ijk}

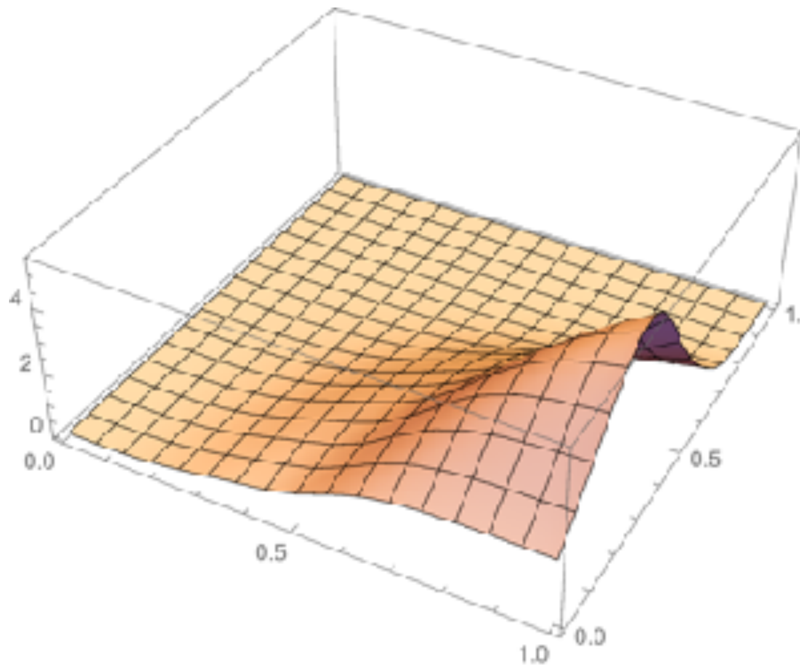


Gluon distribution function

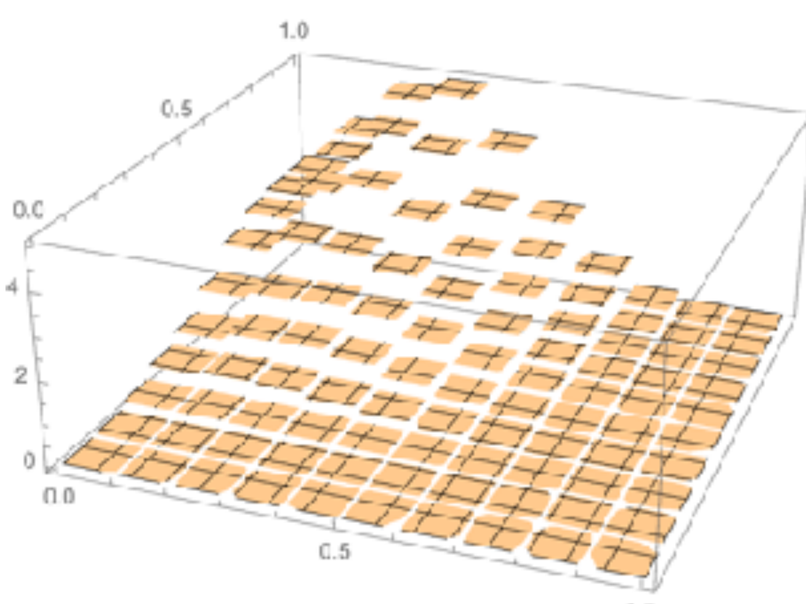
$P_b(x;t,z)$

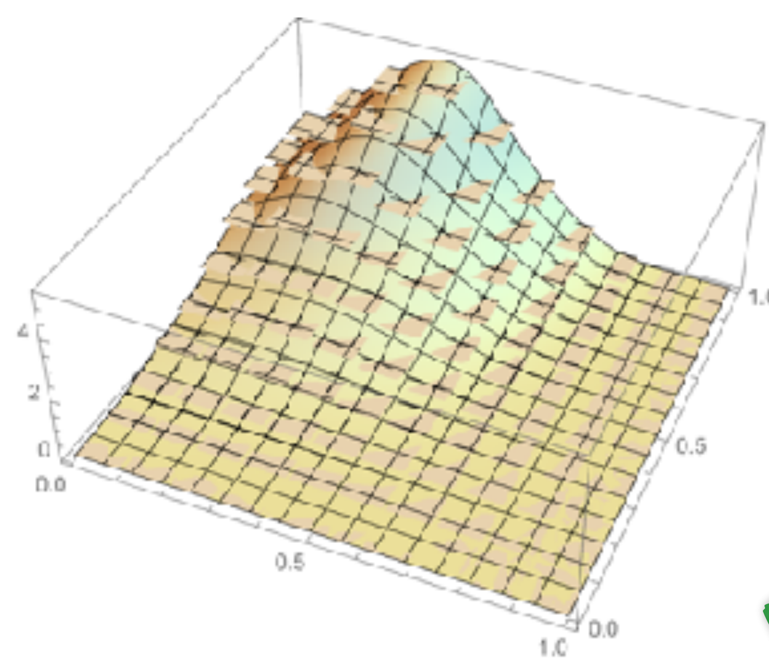


$Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$



- One Grid for the Gluon distribution

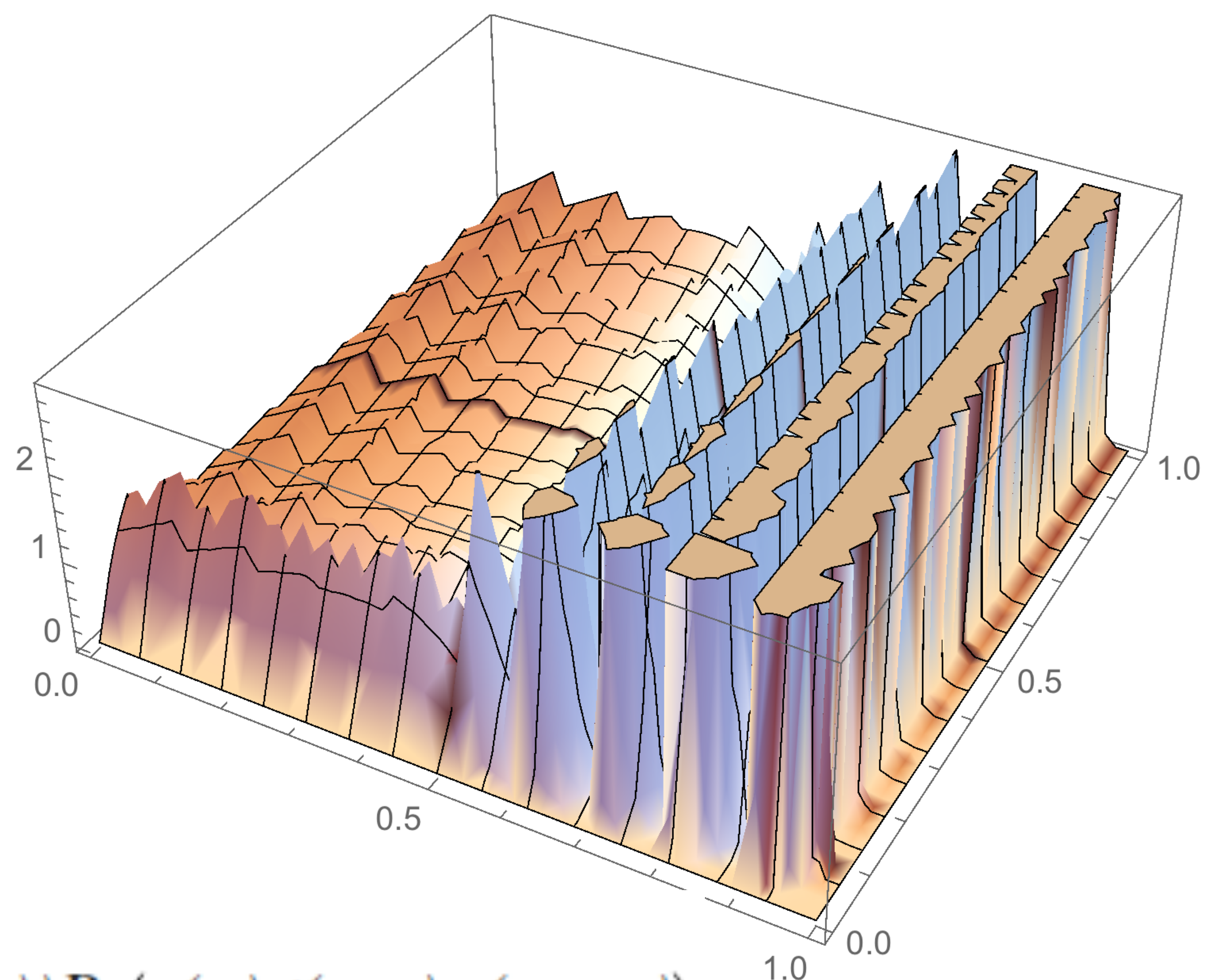




G_{ijk}

C/ Plot the ratio of the two function which should be equal to one with good agreement .

For the small values of the G(s,v,r) the plot was not perfectly fitting, because we took the values in the middle of the tiles , which is very small compared to the continuous function.



$$G_{ijk} \Big/ Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$$

Kinematics Drell Yan (The Jacobean)

Using the four vector and the Jacobean to generate the one body phase space.

$$-1 < \mathrm{Cos}\theta < 1 \quad \Rightarrow \quad \mathrm{Cos}\theta = 1 - 2\,r_1$$

$$0 < \varphi < 2\pi \quad \Rightarrow \quad \varphi = 2\pi r_2$$

$$-\frac{1}{2}\ln\left[\frac{x_1}{x_2}\right] < y < \frac{1}{2}\ln\left[\frac{x_1}{x_2}\right] \quad \Rightarrow \quad -\frac{1}{2}\ln\frac{E_{CM}}{M} < y < \frac{1}{2}\ln\frac{E_{CM}}{M}$$

$$\Rightarrow \quad y = \ln\frac{E_{CM}}{M}\,(2\,r_3 - 1)$$

$$0 < M < E_{CM} \quad \Rightarrow \quad M = r_4\,E_{CM}$$

$$\mathrm{Jac} = \frac{1}{32\,\pi^2}\,\frac{1}{s}\,\frac{dM^2}{dr_M}\,\frac{d\mathrm{Cos}}{dr_\theta}\,\frac{dy}{dr_y}\,\frac{d\varphi}{dr_\varphi}$$

N body phase space

- The generator I have is to produce one body phase space,
- Write a function that takes phi, y, tau, m, costheta, t, z, given those variables I write one point phase space,
- I need to build a recursive generator

$$\sigma_N = \left\langle \int dr_{N-1} \int dr_{rad} \int \frac{d\Phi_{N-1}}{dr_{N-1}} \frac{dt\, dz\, d\varphi}{dr_{rad}} \frac{d\Phi_N}{d\Phi_{N-1}\, dt\, dz\, d\varphi} \frac{d\sigma}{d\Phi_N} \right\rangle$$

Conclusion

•We are aiming to develop a phase space generator that distribute phase space points according to the singular limit of QCD using the distribution functions.

•Create a recursive phase space generator that at each step adds one extra parton to the phase space.

•The main power of this approach comes from choosing the distribution according to the QCD radiation and splitting functions.